

Consistent deformations method applied to a topological coupling of antisymmetric gauge fields in D=3

D. M. Medeiros^{a,b}, R. R. Landim^a and C. A. S. Almeida^{a,1}

^a *Universidade Federal do Ceará - Departamento de Física
C.P. 6030, 60470-455 Fortaleza-Ce, Brazil*

^b *Universidade Estadual do Ceará - Departamento de Física e Química
Av. Paranjana, 1700, 60740-000 Fortaleza-Ce, Brazil*

Abstract

In this work we use the method of consistent deformations of the master equation by Barnich and Henneaux in order to prove that an abelian topological coupling between a zero and a two form fields in D=3 has no nonabelian generalization. We conclude that a topologically massive model involving the Kalb-Ramond two-form field does not admit a nonabelian generalization. The introduction of a connection-type one form field keeps the previous result.

PACS: 11.15.-q, 11.10.Ef, 11.10.Kk

Keywords: method of consistent deformations; nonabelian gauge theories; anti-symmetric tensor gauge fields; BRST/anti-BRST symmetry; topological mass generation

Antisymmetric tensor gauge fields coupled to abelian or nonabelian gauge fields has been studied during the last decade. In particular, four-dimensional BF models has been extensively investigated in different contexts (see Ref. [1] and references therein). Besides supergravity and string theories [2], two form fields play an important role in topological mass generation mechanism to Kalb-Ramond fields [3,4]. Indeed, a nonabelian theory involving an antisymmetric tensor field coupled to a gauge field appears as an alternative mechanism for generating vector bosons masses, similar to the theory of a heavy Higgs particle [5]. Therefore analysis of nonabelian topological terms

¹ Electronic address: carlos@fisica.ufc.br

deserves some attention. It is worth to mention a generalization to a compact nonabelian gauge group of an abelian mechanism in the context of nonabelian quantum hair on black holes [6].

Using the technique of consistent deformation, Henneaux *et al.* [7], have proved that is not possible to generalize the four dimensional topological mass mechanism to its nonabelian counterpart with the same field contents and fulfilling the power-counting renormalization requirements. In this way, they put in more rigorous grounds the need to add an auxiliary field.

Let us begin by presenting a recently proposed abelian three-dimensional action with a topological term involving a two-form gauge field $B_{\mu\nu}$ and a scalar field φ [8]:

$$S_{inv}^A = \int d^3x \left(\frac{1}{12} H_{\mu\nu\alpha} H^{\mu\nu\alpha} + \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi + \frac{m}{2} \epsilon^{\mu\nu\alpha} B_{\mu\nu} \partial_\alpha \varphi \right), \quad (1)$$

where $H_{\mu\nu\alpha}$ is the totally antisymmetric tensor $H_{\mu\nu\alpha} = \partial_\mu B_{\nu\alpha} + \partial_\alpha B_{\mu\nu} + \partial_\nu B_{\alpha\mu}$. The action (1), is invariant under the transformation

$$\delta\varphi = 0, \quad \delta B_{\mu\nu} = \partial_{[\mu} \omega_{\nu]} . \quad (2)$$

The model described by action (1) can be consistently obtained by dimensional reduction of a four-dimensional $B \wedge F$ model if we discard the Chern-Simons-like terms [8].

The purpose of the present note is to prove that there is no power counting renormalizable nonabelian generalization of the action (1). We begin making an analysis of the possibility to construct the nonabelian action with $\varphi \rightarrow \varphi^a$ and $B_{\mu\nu} \rightarrow B_{\mu\nu}^a$, i.e, with the same field content, and the same number of local symmetries, by making use of the method of consistent deformation [9]. As will be proved, there is a no-go theorem for this construction. The same occur with an introduction of a connection-type one form gauge field.

Let us now apply the consistent deformation method described in [9]. We shall start therefore with the following invariant action

$$S'_0 = \int d^3x \left(\frac{1}{12} H_{\mu\nu\alpha}^a H^{a\mu\nu\alpha} + \frac{1}{2} \partial_\mu \varphi^a \partial^\mu \varphi^a \right), \quad (3)$$

where now φ^a and $H_{\mu\nu\alpha}^a$ are scalar fields and the abelian curvature tensor for a set of N fields. All fields are valued in the Lie algebra \mathcal{G} of some Lie group G . Since we are interested if the mass term can exist in a nonabelian extension of (1), the mass parameter will be considered as a deformation parameter. The action (3) is invariant under the transformations

$$\delta\varphi^a = 0, \quad \delta B_{\mu\nu}^a = \partial_\mu \omega_\nu^a - \partial_\nu \omega_\mu^a. \quad (4)$$

Since the transformation of $B_{\mu\nu}^a$ is reducible, we introduce a set of ghosts (η_μ^a, ρ^a) , where η_μ^a is a ghost for the gauge transformation of $B_{\mu\nu}^a$, and ρ^a the ghost for ghost for taking into account this reducibility. For all fields of the model we introduce the corresponding antifields $(B_{\mu\nu}^{*a}, \varphi^{*a}, \eta_\mu^{*a}, \rho^{*a})$. The antifields action reads

$$S'_{ant} = \int d^3x \left(\frac{1}{2} B^{*\mu\nu a} \partial_{[\mu} \eta_{\nu]}^a + \eta^{*\mu a} \partial_\mu \rho^a \right). \quad (5)$$

The free action

$$S_0 = S'_0 + S'_{ant}, \quad (6)$$

is solution of the master equation

$$(S_0, S_0) = 0, \quad (7)$$

with

$$(S_0, S_0) = \int d^3x \left(\frac{\delta S_0}{\delta \varphi^a} \frac{\delta S_0}{\delta \varphi^{*a}} + \frac{1}{2} \frac{\delta S_0}{\delta B^{a\mu\nu}} \frac{\delta S_0}{\delta B_{\mu\nu}^{*a}} + \frac{\delta S_0}{\delta \eta^{a\mu}} \frac{\delta S_0}{\delta \eta_\mu^{*a}} + \frac{\delta S_0}{\delta \rho^a} \frac{\delta S_0}{\delta \rho^{*a}} \right) \quad (8)$$

The nilpotent BRST transformation s on all fields and antifields is

$$\begin{aligned} s\varphi^a &= 0, & s\varphi_a^* &= -\partial^2 \varphi, \\ sB_{\mu\nu}^a &= \partial_\mu \eta_\nu^a - \partial_\nu \eta_\mu^a, & sB_a^{*\mu\nu} &= -\partial_\rho H_a^{\rho\mu\nu}, \\ s\eta_\mu^a &= \partial_\mu \rho^a, & s\eta_a^{*\mu} &= \partial_\rho B_a^{*\rho\mu}, \\ s\rho^a &= 0, & s\rho_a^* &= -\partial_\mu \eta_a^{*\mu}. \end{aligned} \quad (9)$$

We show in the table (1) below, the canonical dimension and the ghost number for all fields and antifields of the model.

Having the ghost number and dimension of all fields and antifields at hand, we are now able to solve our problem using the consistent deformation method. The action (6) will be deformed to a new action S in powers of the deformation parameters:

$$S = S_0 + \sum_i g_i S_i + \sum_{i,j} g_i g_j S_{ij} + \dots, \quad (10)$$

	Φ^a	$B_{\mu\nu}^a$	η_μ^a	ρ^a	Φ^{*a}	$B_{\mu\nu}^{*a}$	η_μ^{*a}	ρ^{*a}
N_g	0	0	1	2	-1	-1	-2	-3
dim	1/2	1/2	-1/2	-3/2	5/2	5/2	7/2	9/2

Table 1

Ghost numbers and dimensions.

where $S_i, S_{ij}..$ are local integrated polynomials with ghost number zero and dimension bounded by three, and g_i are the deformed parameters with non-negative mass dimension. The action (10) must satisfy the master equation

$$(S, S) = 0. \quad (11)$$

Expanding the master equation (11) in powers of the deformation parameters, we have

$$(S_0, S_0) = 0, \quad (12)$$

$$(S_0, S_i) = 0, \quad (13)$$

$$2(S_0, S_{ij}) + (S_i, S_j) = 0. \quad (14)$$

The equation (12) is the master equation for the S_0 , and it does not give any additional information. The equation (13) tell us that S_i has to be a BRST invariant under (9). We must neglect BRST exacts, since this correspond to fields redefinitions. The last equation (14) is satisfied only if the antibracket (S_i, S_j) is a trivial cocycle.

Let us now construct all S_i solution of equation (13). First we focus our attention to terms that do not deform the gauge symmetry, i.e, terms constructed with the fields only. Due to trivial BRST transformation of φ^a , the all possible terms with this field are

$$S_1 = \int d^3x (\alpha_a \varphi_a), \quad S_2 = \int d^3x (\alpha_{ab} \varphi_a \varphi_b), \quad (15)$$

$$S_3 = \int d^3x (\alpha_{abc} \varphi_a \varphi_b \varphi_c), \quad S_4 = \left(\int d^3x \alpha_{abcd} \varphi_a \varphi_b \varphi_c \varphi_d \right), \quad (16)$$

$$S_5 = \int d^3x (\alpha_{abcde} \varphi_a \varphi_b \varphi_c \varphi_d \varphi_e), \quad S_6 = \int d^3x (\alpha_{abcdef} \varphi_a \varphi_b \varphi_c \varphi_d \varphi_e \varphi_f), \quad (17)$$

where α 's are parameters. The most general invariant local integrable terms that can be constructed with $B_{\mu\nu}^a$ and φ^a mixed are

$$S_7 = \int d^3x (m_{ab} \epsilon^{\mu\nu\alpha} H_{a\mu\nu\alpha} \varphi_b), \quad S_8 = \int d^3x (m_{abc} \epsilon^{\mu\nu\alpha} H_{a\mu\nu\alpha} \varphi_b \varphi_c) \quad (18)$$

$$S_9 = \int d^3x \ (m_{abcd} \epsilon^{\mu\nu\alpha} H_{a\mu\nu\alpha} \varphi_b \varphi_c \varphi_d), \quad (19)$$

with m_{ab} having dimension of mass, m_{abc} of dimension 1/2 and m_{abcd} as a dimensionless parameter.

Observing the table (1), it is easy to see that it is impossible to construct invariant local integrated polynomials with dimension bounded by three with the antifields. This means that the algebra of the gauge symmetry is undeformed, i.e., we do not have a nonabelian generalization of the action (1), the only possibility being with an introduction of extra fields or non-renormalizable couplings.

Let us now introduce a set of abelian vectorial gauge field in order to implement the possible nonabelian generalization of (1). We take the mass dimension of all vector fields equal to one. Therefore, those fields assume a non-dynamical character. The BRST transformations are

$$sA_\mu^a = \partial_\mu c^a, \quad sc^a = 0, \quad (20)$$

where c^a are the ghost for the abelian transformation of A_μ^a . We must add to the action (6) the corresponding antifield action

$$S''_{ant} = \int d^3x \ A_{a\mu}^* \partial^\mu c_a. \quad (21)$$

The new antifields have the following BRST transformations

$$sA_\mu^{*a} = 0, \quad sc^{*a} = \partial_\mu A^{*a\mu}. \quad (22)$$

We show in the table (2) below the ghost number and dimension for the new fields (antifields).

	A_μ^a	c^a	A_μ^{*a}	c^{*a}
N_g	0	1	-1	-2
dim	1	0	2	3

Table 2

Ghost numbers and dimensions.

The all possible invariant integrated local polynomials that can be constructed with all fields and antifields are

$$S_{10} = g \int d^3x \ f_{abc} (\varphi_a^* \varphi_b c_c - \partial^\mu \varphi_a \varphi_b A_{c\mu}), \quad (23)$$

$$S_{11} = \mu_{ab} \int d^3x \ (A_{a\mu}^* \eta_b^\mu - c_a^* \rho_b), \quad (24)$$

$$S_{12} = h \int d^3x \, k_{abc} \left(A_a^* A_b^\mu c_c - \frac{1}{2} c_a^* c_b c_c \right), \quad (25)$$

where g, h are dimensionless parameter, μ is a matrix with dimension $3/2$, and $f_{abc}(k_{abc})$ are dimensionless parameters antisymmetric in its first(last) two indices. Now we perform the calculation of the antibrackets (S_i, S_j) , with $i, j = 1, 2, \dots, 12$, in order to fit the second order consistency condition. As we have already seen above, these antibrackets must be a BRST exact. The antibrackets (S_m, S_n) , for $n, m = 1, 2, \dots, 9$ are identically zero, due to absence of antifields in $S_n, n = 1, 2, \dots, 9$. The antibracket (S_{10}, S_{10}) is

$$\begin{aligned} (S_{10}, S_{10}) = & g^2 \int d^3x \, f_{abc} f_{ab'c'} \left(\varphi_{b'}^* \varphi_b c_c c_{c'} + \varphi_{[b} \partial_\mu \varphi_{b']} A_{c'}^\mu c_c \right) \\ & - \frac{g^2}{2} s \left(\int d^3x \, f_{abc} f_{ab'c'} \varphi_b \varphi_{b'} A_{c\mu} A_{c'}^\mu \right), \end{aligned} \quad (26)$$

where, $\varphi_{[b} \partial_\mu \varphi_{b']} = \varphi_b \partial_\mu \varphi_{b'} - \varphi_{b'} \partial_\mu \varphi_b$. The first term in (26), is not a BRST trivial and it could jeopardize the nonabelian implementation. In order to circumvent this, we must have the identification $h k_{abc} = g f_{abc}$, and f_{abc} being the structure constant of a Lie group. Therefore the S_{10} and S_{12} are replaced by the sum

$$S'_{10} = g \int d^3x \, f^{abc} \left(\varphi^{*a} \varphi^b c^c - \partial^\mu \varphi^a \varphi^b A_\mu^c + A^{*a} A^{b\mu} c^c - \frac{1}{2} c^{*a} c^b c^c \right). \quad (27)$$

It is easy to see that now (S'_{10}, S'_{10}) is BRST trivial

$$(S'_{10}, S'_{10}) = -\frac{g^2}{2} s \left(\int d^3x \, f_{abc} f_{ab'c'} \varphi_b \varphi_{b'} A_{c\mu} A_{c'}^\mu \right). \quad (28)$$

The antibrackets (S'_{10}, S_n) , with $n = 1, 2, \dots, 6$, give us constraint for the parameters α : $\alpha_a = \alpha_{abc} = \alpha_{abcde} = 0$, $\alpha_{ab} = a_1 \delta_{ab}$, $\alpha_{abcd} = a_2 \delta_{ab} \delta_{cd}$, $\alpha_{abcdef} = a_3 \delta_{ab} \delta_{cd} \delta_{ef}$, i.e., only the terms $\varphi^2 = \varphi_a \varphi_a$, $(\varphi^2)^2$ and $(\varphi^2)^3$ are permitted. The last antibrackets reads

$$(S_{11}, S_{11}) = 0,$$

$$\begin{aligned} (S'_{10}, S_{11}) = & g \int d^3x \, f_{abc} \mu_{ab'} \left(\rho_{b'} \varphi_b^* \varphi_c + \rho_{b'} A_{b\mu}^* A_c^\mu - \rho_{b'} c_b^* c_c \right. \\ & \left. - \eta_{b'}^\mu \partial_\mu \varphi_b \varphi_c - \eta_{b'}^\mu A_{b\mu}^* c_c \right), \end{aligned} \quad (29)$$

$$(S'_{10}, S_7) = g \int d^3x \, f_{abc} m_{b'a} \varepsilon_{\mu\nu\alpha} H_{b'}^{\mu\nu\alpha} \varphi_b c_c,$$

$$(S'_{10}, S_8) = g \int d^3x \, f_{abc} (m_{b'c'a} + m_{b'ac'}) \varepsilon_{\mu\nu\alpha} H_{b'}^{\mu\nu\alpha} \varphi_b \varphi_{c'} c_c,$$

$$(S'_{10}, S_9) = g \int d^3x f_{abc}(m_{b'c'd'a} + m_{b'c'ad'} + m_{b'ac'd'})\varepsilon_{\mu\nu\alpha}H_{b'}^{\mu\nu\alpha}\varphi_b\varphi_{c'}\varphi_{d'}c_c.$$

The last four antibrackets are not BRST trivial, representing thus an obstruction to the deformation of the master equation. The only way to remedy this is setting $g = 0$, or setting $S_7 = S_8 = S_9 = S_{11} = 0$. In the case $g = 0$ we have lost the deformation of the abelian algebra, i.e, we have a set of abelian fields not representing a nonabelian generalization of (1). In the case in which $S_7 = 0$, we have lost the mass generation of the model. We have thus proved that there are no nonabelian generalization of the action (1), even with an addition of an auxiliary vector gauge field.

It is interesting to remark that the introduction of an one form gauge connection A is required to go further in the nonabelian generalization of our model (1), although our original abelian action (1) does not contain this field.

We can quote some works that built some kind of nonabelianization of the model under consideration. However, the work of Oda and Yahikozawa [10] as well as the work of Smailagic and Spallucci [11], have considered the connection one form A as a flat background field (so imposing an extra constraint in the model). On the other hand, Del Cima *et al.* [12], have studied the finiteness of a three-dimensional extension of the BF model, called BFK model, which does not have kinetic term for the antisymmetric gauge field (this term violate the nonabelian gauge invariance which is straightforwardly implemented for vectorial fields).

Motivated by possible nonabelian topological mass generation for a Kalb-Ramond field in three dimensions, we have considered deformations of a model involving a topological coupling between a second rank antisymmetric tensor field and a scalar field, using the method of consistent deformations. But an obstruction arise, leading us to a no-go theorem, namely, if we require power-counting renormalizable couplings and the same field content, the nonabelian extension for the model is forbidden.

ACKNOWLEDGMENTS

We would like to thank Dr. O. S. Ventura for helpful discussions. This work was supported in part by Fundação Cearense de Amparo à Pesquisa-FUNCAP.

References

- [1] D. Birmingham, M. Blau, M. Rakowski and G. Thompson, *Phys. Rep.* **209** (1991) 129.

- [2] M. Green, J. Schwarz and E. Witten, Superstring Theory, 2 volumes (Cambridge University Press, Cambridge, 1987).
- [3] T. J. Allen , M. J. Bowick and A. Lahiri, *Mod. Phys. Lett.* **A6** (1991) 559; D. S. Hwang and C. Y. Lee, *J. Math. Phys.* **38** (1) (1997) 30; A. Lahiri, *Phys. Rev.* **D 55** (1997) 5045; A. Smailagic and E. Spallucci, *Phys. Rev.* **D61** (2000) 067701.
- [4] R. R. Landim and C. A. S. Almeida, Topologically massive nonabelian BF models in arbitrary space-time dimensions, hep-th/0010050, to appear in *Phys. Lett. B*.
- [5] D.Z. Freedman and P.K. Townsend, *Nucl. Phys.* **B177** (1981) 282; T.E. Clark, C.H. Lee, and S.T. Love, *Nucl. Phys.* **B308** (1988) 379; S.-J. Rey, *Phys. Rev.* **D40** (1989) 3396; M. LeBlanc, R. MacKenzie, P.K. Panigrahi, and R. Ray, *Int. J. Mod. Phys.* **A9** (1994) 4717.
- [6] A. Lahiri, *Phys. Lett.* **B 297** (1992) 248.
- [7] M. Henneaux, V. E. R. Lemes, C. A. G. Sasaki, S. P. Sorella, O. S. Ventura, L. C. Q. Vilar, *Phys. Lett.* **B410** (1997) 195.
- [8] D. M. Medeiros, R. R. Landim, C. A. S. Almeida, *Europhys. Lett.* **48** (1999) 610.
- [9] G. Barnich and M. Henneaux, *Phys. Lett.* **B311** (1993) 123.
- [10] I. Oda and S. Yahikozawa, *Phys. Lett.* **B234** (1990) 69.
- [11] A. Smailagic and E. Spallucci, *Phys. Lett.* **B489** (2000) 435.
- [12] O. M. Del Cima, J. M. Grimstrup, M. Schweda, *Phys. Lett.* **B463** (1999) 48.